Name ______________________________ Section __________ Date __________

# 2 Scientific Measurement

## Skillsheet

### Exponents

Exponents are widely used in science. They are a convenient means to express large and small numbers. An exponent is a number that indicates how many times another number should be multiplied by itself. To illustrate exponents and how to work with them, many examples are presented. Examine each carefully.

If five is multiplied by itself 3 times, $5 \times 5 \times 5$, the result is 125. This may be written using exponents as: $5^3 = 125$. Here, 3 is the exponent, and 5 the base.

Consider a fraction with an exponent.

$$\left(\frac{1}{4}\right)^3 = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$$

Another way to express $\left(\frac{1}{4}\right)^3$ is to reverse the fraction and change the sign of the exponent.

$$\left(\frac{1}{4}\right)^3 = \frac{1}{4^3} = 4^{-3} = \frac{1}{64} \text{ as before.}$$

$$5^{-1} = \frac{1}{5} \text{ Here } -1 \text{ is the exponent.}$$

Ten is often used as a base with an exponent. Note the patterns in the following set of examples. Starting with the middle line, read up and down as well as across.

$$\begin{aligned}
10^{-3} &= \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} &&= \frac{1}{1000} &&= .001 \\
10^{-2} &= \frac{1}{10} \times \frac{1}{10} &&= \frac{1}{100} &&= .01 \\
10^{-1} &= \frac{1}{10} &&= \frac{1}{10} &&= .1 \\
10^{0} &= 1 &&= 1 &&= 1 \\
10^{1} &= 10 &&= 10 &&= 10 \\
10^{2} &= 10 \times 10 &&= 100 &&= 100 \\
10^{3} &= 10 \times 10 \times 10 &&= 1000 &&= 1000
\end{aligned}$$

Numbers with exponents can be added, subtracted, multiplied, divided, raised to powers, their roots taken, etc. Consider a number multiplied by another number with an exponent.

$$3 \times 2^4 = 3 \times 2 \times 2 \times 2 \times 2 = 3 \times 16 = 48$$

$$5.1 \times 10^3 = 5.1 \times 10 \times 10 \times 10 = 5.1 \times 1000 = 5100$$

The number in front of ten is called the coefficient; in this case, it is 5.1.

A number can be represented in many ways by correctly changing *both* the exponent of ten and the position of the decimal point. For example:

$$\begin{aligned}
54\,321 &= 54.321 \times 10^3 &&= 543.21 \times 10^2 \\
&= 5432.1 \times 10^1 &&= 54\,321 \times 10^0 \\
&= 543\,210 \times 10^{-1} &&= 5\,432\,100 \times 10^{-2}
\end{aligned}$$

A number written in this manner is in exponential form. If this form has only one digit in front of the decimal, it is said to be in "scientific notation."

To add or subtract numbers in exponential form, the base *and* the exponents must be the same. In the examples below, the base is 10 and the exponent is 2.

$$4 \times 10^2 + 3 \times 10^2 = 7 \times 10^2$$

Notice that $10^2$ is found in both numbers as well as in the answer. It may be read as: $400 + 300 = 700$. Subtraction is very similar.

$$4 \times 10^2 - 3 \times 10^2 = 1 \times 10^2 = 10^2 = 100$$

This may be read as: $400 - 300 = 100$. Below, the coefficients cannot be added directly until one of the numbers is changed so that its exponent is the same as the other. Notice for:

$5 \times 10^3 + 4 \times 10^2$, the exponents aren't equal.

$5 \times 10^3 = 50 \times 10^2$, so the problem becomes:

$$50 \times 10^2 + 4 \times 10^2 = 54 \times 10^2 = 5.4 \times 10^3.$$

This may be written as: $5000 + 400 = 5400$, but this "long" method is inconvenient when numbers are very large or very small. Subtraction is similar:

$5 \times 10^3 - 4 \times 10^2$ becomes:

$$50 \times 10^2 - 4 \times 10^2 = 46 \times 10^2 = 4.6 \times 10^3$$

Multiplying numbers with exponents is easier than addition or subtraction. The bases *must* be the same, as is usually the case.

$$4 \times 10^2 \times 6 \times 10^5 = 24 \times 10^7 = 2.4 \times 10^8$$

Notice, to obtain 24, the coefficients of 4 and 6 were multiplied together. To obtain the exponent of 7, the exponents of 2 and 5 were *added*. Other examples:

$8 \times 10^4 \times 2 \times 10^{-3} = 16 \times 10^1 = 1.6 \times 10^2 = 160$

$8 \times 10^{-4} \times 2 \times 10^3 = 16 \times 10^{-1} = 1.6 \times 10^0 = 1.6$

$8 \times 10^{-4} \times 2 \times 10^{-3} = 16 \times 10^{-7} = 1.6 \times 10^{-6} = .0000016$

In the division process for numbers with exponents, the coefficients are divided and the exponents are *subtracted* (the denominator from the numerator). Here are some examples.

$$8 \times 10^9 \div 2 \times 10^3 = \frac{8}{2} \times 10^{(9-3)} = 4 \times 10^6$$

$$8 \times 10^9 \div 2 \times 10^{-3} = \frac{8}{2} \times 10^{(9--3)} = 4 \times 10^{12}$$

$$8 \times 10^{-9} \div 2 \times 10^3 = \frac{8}{2} \times 10^{(-9-3)} = 4 \times 10^{-12}$$

The next division examples are in a different form, and the numbers are more difficult. Work them on your own and compare your answers with those given.

$$\frac{6 \times 10^{-5}}{8 \times 10^2} = \frac{6}{8} \times 10^{[-5-(+2)]} = .75 \times 10^{-7} = 7.5 \times 10^{-8}$$

$$\frac{2.8 \times 10^{-4}}{4.0 \times 10^{-13}} = \frac{2.8}{4.0} \times 10^{[-4-(-13)]} = .7 \times 10^9 = 7.0 \times 10^8$$

Numbers in exponential form can be raised to powers and also have their roots calculated.

$$(4 \times 10^2)^3 = 4^3 \times (10^2)^3 = 4 \times 4 \times 4 \times 10^2 \times 10^2 \times 10^2 = 64 \times 10^6$$

The coefficient, 4, was cubed as was $10^2$. The exponent, 6, can be obtained by multiplying the exponent, 2, by the power, 3. Another example:

Squaring $5 \times 10^4$ appears as $(5 \times 10^4)^2$.

This is: $5 \times 5 \times 10^4 \times 10^4$, which equals $25 \times 10^8$.

The square root of $25 \times 10^8 = \sqrt{(25 \times 10^8)} = \sqrt{25} \times \sqrt{(10^8)} = \sqrt{25} \times (10^8)^{\frac{1}{2}}$.

The square root of 25 is 5, and $8 \times \frac{1}{2} = 4$.

The square root of $25 \times 10^8 = 5 \times 10^4$.

It may be necessary to adjust the number so that its exponent of ten is evenly divisible by the root:

$\sqrt{160 \times 10^5}$ is the same as $\sqrt{16 \times 10^6}$.

$\sqrt{16} = 4$ and $\sqrt{10^6} = 10^3$.

The square root of $160 \times 10^5 = 4 \times 10^3$.

## Practice Problems

1. a. $0.00001 =$ ______

   b. $10\,000 =$ ______

   c. $4 \times 3^2 =$ ______

   d. $4 \times 10^3 =$ ______

2. a. $2 \times 10^5 + 6 \times 10^5 =$ ______

   b. $5 \times 10^3 + 2 \times 10^3 =$ ______

   c. $5 \times 10^4 + 2 \times 10^3 =$ ______

3. a. $6 \times 10^5 - 2 \times 10^5 =$ ______

   b. $5 \times 10^3 - 2 \times 10^3 =$ ______

   c. $5 \times 10^4 - 2 \times 10^3 =$ ______

   d. $2 \times 10^5 - 6 \times 10^5 =$ ______

4. a. $2 \times 10^5 \times 3 \times 10^5 =$ ______

   b. $2 \times 10^3 \times 6 \times 10^2 =$ ______

   c. $2 \times 10^{-5} \times 3 \times 10^8 =$ ______

   d. $2 \times 10^{-5} \times 3 \times 10^{-8} =$ ______

5. a. $6 \times 10^7 \div 2 \times 10^4 =$ ______

   b. $6 \times 10^7 \div 2 \times 10^{-4} =$ ______

   c. $6 \times 10^{-7} \div 2 \times 10^{-4} =$ ______